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Interest Rate Sensitivity

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Introduction

The interest rate sensitivity widget and report estimate how sensitive a portfolio's fixed income holdings are to changes in interest rates. Interest rate sensitivity is a measure of how much the price of a fixed income holding will fluctuate due to changes in the interest rate environment. Holdings that are more sensitive may have greater price fluctuations than those with less sensitivity.

Bond Example

For each calculation, a hypothetical bond with the below characteristics will be used.

Face Value:	$1,000
Current Market Price:	$973.3569
Coupon Rate:	6% paid semi-annually ($30 per payment)
Current YTM:	7%
Maturity:	3 years (6 semi-annual periods)
Issue Date:	January 15, 2023
Maturity Date:	January 15, 2026

Macaulay Duration

Measures in years the weighted average time until a bond's cash flows are received, reflecting the bond's sensitivity to interest rate changes. Macaulay duration quantifies the time it takes for an investor to be repaid through a combination of interest and principal payments.

Macaulay Duration Formula

$$Macaulay\ Duration = \frac{\sum_{i=1}^{n} PV(i) \times \frac{t \times CF(i)}{P}}{P}$$

Where:

PV =	Present value of each cash flow $= CF/(1+r)^t$
t =	Time period of each cash flow (in years)
CF =	Cash flow amount (coupon payment or coupon + face value)
P =	Current bond price
r =	Yield to maturity per period ($YTM/2$ for semi-annual bonds)
n =	Total number of periods

Macaulay Duration Example

$$i = 1: [(30/1.035^1) \times (0.5/973.3569)] \times 973.3569 = 14.492$$

$$i = 2: [(30/1.035^2) \times (1.0/973.3569)] \times 973.3569 = 28.0053$$

$$i = 3: [(30/1.035^3) \times (1.5/973.3569)] \times 973.3569 = 40.5874$$

$$i = 4: [(30/1.035^4) \times (2.0/973.3569)] \times 973.3569 = 52.2865$$

$$i = 5: [(30/1.035^5) \times (2.5/973.3569)] \times 973.3569 = 63.1479$$

$$i = 6: [(1030/1.035^6) \times (3.0/973.3569)] \times 973.3569 = 2{,}513.7169$$

$$Macaulay\ Duration = Sum\ of\ all\ terms/P = 2{,}712.2367/973.3569 = 2.7865\ years$$

Modified Duration

Measures the percentage change in a bond's price for a 1% (100 basis point) change in yield. It represents the slope of the price-yield relationship at a given point.

Modified Duration Formula

$$Modified\ Duration = \frac{Macaulay\ Duration}{1 + \left(\frac{YTM}{n}\right)}$$

Where:

YTM = Yield to maturity
n = Total number of coupon periods per year

Modified Duration Example

$$Modified\ Duration = \frac{2.7865}{1 + \left(\frac{0.07}{2}\right)} = \frac{2.7865}{1.035} = 2.6922$$

Convexity

It measures the curvature of the relationship between bond prices and yields. Convexity demonstrates why Modified Duration alone (which assumes a linear relationship) becomes less accurate for larger interest rate changes. Applying Convexity to the calculated Modified Duration makes price estimates more accurate, especially for larger yield changes.

Convexity Formula

$$Convexity \approx \frac{P(+) + P(-) - 2P(0)}{P(0) \times \Delta y^2}$$

Where:

P(+) = Bond market value after yield increase
P(-) = Bond market value after yield decrease
P(0) = Current bond market value
Δy = Change in yield (typically 0.0001 or 1 basis point)

Convexity Example

$P(+)$ calculation using $\Delta y = 0.01\%$ resulting in a current YTM of 3.51% $((7.0\%/2) + 0.01\%)$ per payment period.

$$P(+) = \frac{30}{1.0351^1} + \frac{30}{1.0351^2} + \frac{30}{1.0351^3} + \frac{30}{1.0351^4} + \frac{30}{1.0351^5} + \frac{1030}{1.0351^6}$$

$$P(+) = 28.9827 + 27.9999 + 27.0504 + 26.1331 + 25.2469 + 837.4200$$

$$P(+) = 972.833$$

$P(-)$ calculation using $\Delta y = -0.01\%$ resulting in a current YTM of 3.49% $((7.0\%/2) - 0.01\%)$ per payment period.

$$P(-) = \frac{30}{1.0349^1} + \frac{30}{1.0349^2} + \frac{30}{1.0349^3} + \frac{30}{1.0349^4} + \frac{30}{1.0349^5} + \frac{1030}{1.0349^6}$$

$$P(-) = 28.9883 + 28.0107 + 27.0661 + 26.1533 + 25.2714 + 838.3915$$

$$P(-) = 973.8813$$

Calculating Convexity using the $P(+)$ and $P(-)$ values above.

$$Convexity \approx \frac{972.833 + 973.8813 - 2(973.3569)}{973.3569 \times 0.0001^2}$$

$$Convexity \approx \frac{0.0005}{0.00000973}$$

$$Convexity \approx 51.3874$$

Estimating Bond Value Change

For a given rate change, both the Duration Effect and Convexity Effect can be used to more accurately estimate how the value of a fixed income holding will be impacted.

Bond Value Change Formula

$$Duration\ Effect = P(0)\ \times\ (Modified\ Duration\ \times\ -\Delta y)$$

$$Convexity\ Effect = P(0)\ \times\ \left(\frac{1}{2} \times\ Convexity\ \times\ \Delta y^2\right)$$

$$\Delta P\ = Duration\ Effect + Convexity\ Effect$$

Where:

P(0) = Current bond market value

Δy = Change in interest rate

ΔP = Total bond value change

Bond Value Change Example

Current bond value $P(0)$ is $973.36. See below ΔP calculation assuming a +2% rate change.

$$Duration\ Effect = 973.3569\ \times\ (2.6922\ \times\ -0.02)$$

$$Duration\ Effect =\ -52.4094$$

$$Convexity\ Effect = 973.3569\ \times\ \left(\frac{1}{2} \times\ 51.3874\ \times\ 0.02^2\right)$$

$$Convexity\ Effect = 10.0036$$

$$\Delta P\ = -52.4094 + 10.0036$$

$$\Delta P\ = -42.4058 = -\$42.41$$

Rate Change	Duration Effect	Convexity Effect	ΔP	Est. Bond Value	% Value Change
+2%	-$52.41	$10.00	-$42.41	$930.95	-4.36%
0%	$0.00	$0.00	$0.00	$973.36	0.0%
-2%	$52.41	$10.00	$62.41	$1,035.77	6.41%

Conclusion

Within the Interest Rate Sensitivity report and widget, we calculate the Macaulay Duration for each fixed income holding in a portfolio. Using Macaulay Duration, we calculate the Modified Duration which is used to determine the Duration Effect from a given interest rate change. We then estimate each bond position's Convexity by calculating the bond value given a small positive and negative interest rate change. Using a bonds calculated Convexity allows us to determine the Convexity Effect for a given interest rate change. The total estimated bond value change for a given interest rate change is then calculated by combining the Duration Effect and the Convexity Effect demonstrating a position or portfolio's interest rate sensitivity.

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